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Superstrings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ as a coset sigma-model

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Abstract: According to the recent proposal by Aharony, Bergman, Jafferis and Maldacena the $\mathcal{N} = 6$ supersymmetric Chern-Simons theory in three dimensions has a 't Hooft limit whose holographic dual is described by type IIA superstings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background. We argue that the Green-Schwarz action for type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ with κ-symmetry *partially fixed* can be understood as a coset sigma-model on the same space supplied with a proper Wess-Zumino term. We construct the corresponding sigma-model Lagrangian and show that it is invariant under a local fermionic symmetry which for generic bosonic string configurations allows one to remove 8 out of 24 fermionic degrees of freedom. The remaining 16 fermions together with their bosonic partners should describe the physical content of $\mathrm{AdS}_4 \times \mathbb{CP}^3$ superstring. As further evidence, we demonstrate that in the plane-wave limit the quadratic action arising from our model reproduces the one emerging from the type IIA superstring. The coset sigma-model is classically integrable which opens up the possibility to investigate its dynamics in a way very similar to the case of $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstrings.

Keywords: AdS-CFT Correspondence, Sigma Models, Penrose limit and pp-wave background.

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1. Introduction and summary

Superconformal Chern-Simons theories [1] conjectured to describe the low-energy world-volume dynamics of multiple M2-branes are receiving nowadays considerable attention [2]. Recently Aharony, Bergman, Jafferis and Maldacena (ABJM) proposed a new example of the AdS/CFT duality which involves the $\mathcal{N}=6$ superconformal $\mathrm{SU}(N)\times\mathrm{SU}(N)$ Chern-Simons theory in three dimensions and the M-theory on $\mathrm{AdS}_4\times\mathrm{S}^7/\mathbb{Z}_k$, where k is the level of the Chern-Simons action [3].

The ABJM model is characterized by two parameters — the rank N of the two gauge groups $\mathrm{SU}(N)$ and the integer level k which is opposite for the gauge groups. Remarkably, there exists an analogue of the 't Hooft limit, where $N,k\to\infty$ with the ratio $\lambda=2\pi^2 N/k$ kept fixed. In this limit λ becomes continuous allowing therefore for application of standard perturbative techniques. It turns out that at leading order in the weak coupling expansion the corresponding dilatation operator can be identified with an integrable Hamiltonian of the SU(4) spin chain with spins alternating between fundamental and anti-fundamental representations [4]. The set of emerging Bethe equations admits an extension to the full superconformal group $\mathrm{OSP}(2,2|6)$.

According to [3], at strong coupling, i.e. when λ becomes large, the M-theory on $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathbb{Z}_k$ can be effectively described by type IIA superstring theory on the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background. To understand this new example of holography, one needs, therefore, to determine the spectrum of the corresponding string theory. Obviously, the classical bosonic string theory can be formulated as a sigma-model on the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ space. This model is integrable but quantum corrections related to $\mathbb{CP}^3$ are known to spoil its classical integrability [5]. One may hope, however, that inclusion of type IIA fermions in the full model would maintain integrability at the quantum level. This is not an easy question to answer. The presence of the background RR fields sustaining the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ metric [6] suggests to use the Green-Schwarz formulation for type IIA superstrings. On the other hand, the complete Green-Schwarz action on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ to all orders in fermionic variables is unknown.[1] Even the knowledge of the action alone would be of little use to expose integrable properties of the corresponding model.

In this paper we propose a novel way to investigate the dynamics of type IIA strings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ and, in particular, to reveal its classical integrability. The main idea is to follow closely the case of type IIB superstrings on $\mathrm{AdS}_5 \times \mathrm{S}^5$, where the coset sigma-model formulation [8] provides an alternative to the conventional Green-Schwarz approach. The $\mathrm{AdS}_4 \times \mathbb{CP}^3$ space is a coset $\mathrm{SO}(3,2)/\mathrm{SO}(3,1) \times \mathrm{SO}(6)/\mathrm{U}(3)$. The group $\mathrm{SO}(3,2) \times \mathrm{SO}(6)$ is a bosonic subgroup of the superconformal group $\mathrm{OSP}(2,2|6)$ which naturally suggests to include fermionic degrees of freedom by considering a sigma-model on the coset space $\mathrm{OSP}(2,2|6)/(\mathrm{SO}(3,1) \times \mathrm{U}(3))$.

A problem one immediately faces with this formulation is that the corresponding coset space contains 24 real fermions, which is too little in comparison to 32 fermions of the Green-Schwarz type IIA superstring. On the other hand, because of κ-symmetry only half of fermions are physical in the latter case. Thus, the sigma-model we propose could be just a partially κ-symmetry fixed version of the Green-Schwarz type IIA superstring, where only 8 out of 32 fermions have been gauged away. To justify this interpretation, the sigma-model in question when supplied with a proper Wess-Zumino term must allow for a local fermionic symmetry which removes another 8 unphysical fermions. Construction of κ-symmetry transformations which precisely do this job is one of the results of our paper.

We find that for generic bosonic configurations, i.e. the ones, which involve string motion in both AdS_4 and $\mathbb{CP}^3$ directions, the rank of κ-symmetry variations is 8. There are, however, "singular" configurations corresponding to string moving in the AdS part of the coset only. For these configurations the rank of κ-symmetry variations is 12. We argue that the singular nature of these string backgrounds is due to their incompatibility with the κ-symmetry gauge choice that has to be made in order to reduce the full-fledged Green-Schwarz superstring to our coset model.

To get more evidence to our interpretation, we further derive the quadratic fermionic action arising in the expansion of the full sigma-model action around the point-particle geodesics and show that it precisely coincides with the one which emerges from Penrose

[1] In principle, one can obtain the corresponding string action by performing the double dimensional reduction of the supermembrane action on $\mathrm{AdS}_4 \times \mathrm{S}^7$ constructed in [7].

limit of type IIA superstings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ [9]–[12].

It should be noted that our sigma-model construction is very close to that for $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstrings. This allows us to conclude straightforwardly on classical integrability of the model by exhibiting the same type of the Lax connection as was found for $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [13]. We also verify that this Lax connection is compatible with κ-symmetry we found. This opens up the possibility to investigate (partially κ-gauge fixed) strings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ by employing the methods built up for the $\mathrm{AdS}_5 \times \mathrm{S}^5$ case. In particular, imposing the uniform light-cone gauge [14, 15] one can develop semiclassical quantization to verify whether classical integrability is preserved by leading quantum corrections. Then, the subalgebra $\mathfrak{su}(2|2)$ of the global symmetry algebra $\mathrm{OSP}(2,2|6)$ which leaves the light-cone Hamiltonian invariant in the limit of infinite light-cone momentum undergoes a central extension by the generator P of the world-sheet momentum [16, 17]. Assuming integrability of the quantum sigma-model, it would be interesting to see to which extend this symmetry of the light-cone Hamiltonian can be used to fix the form of the scattering matrix. Finite-gap solutions (including fermionic excitations) [18] could be also investigated with the goal of reconstructing the data of the string S-matrix undetermined by symmetries, e.g. the dressing phase [19]. In the $\mathrm{AdS}_5 \times \mathrm{S}^5$ case the dressing phase satisfies the crossing symmetry equation [20], which essentially determines its form [21]. It would be also interesting to understand constraints on the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ scattering matrix imposed by crossing symmetry.

The paper is organized as follows. In the next section after a brief introduction to the Lie algebra $\mathfrak{osp}(2,2|6)$ we present the Lagrangian and equations of motion of the coset model. In section 3 we deduce the corresponding κ-symmetry transformations and analyse the rank of on-shell κ-symmetry transformations. In section 4 we exhibit the Lax connection for our model and demonstrate that under κ-variations it retains on-shell zero curvature. Section 5 is devoted to analysis of the quadratic action for fermions around a null geodesics. Some technical details are relegated to appendices A and B.

2. Sigma-model Lagrangian

2.1 Coset model and its relation to IIA superstrings

To describe superstrings propagating in the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background, one may try to develop the corresponding Green-Schwarz formalism [22]. We recall that the Green-Schwarz superstring involves two Majorana-Weyl fermions of different chirality with the total number of 32 fermionic degrees of freedom. On the other hand, the Green-Schwarz string action exhibits a local fermionic symmetry (κ-symmetry), which allows one to remove a half of them. The remaining 16 fermions are physical and in the light-cone gauge they match with 8 bosons rendering the space-time supersymmetry manifest. Unfortunately, the explicit form of the type IIA Green-Schwarz action in an arbitrary background is known up to quartic terms only [23] and, for this reason, it remains unknown for the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background. Even if such an action would be found, it would not be straightforward to reveal its integrable properties. A great advantage of type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is that it admits an alternative description as a coset sigma-model [8] which allows one, in particular, to prove its classical integrability [13].

To make a progress in understanding the string dynamics in the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ space-time, we devise an approach which does not rely on the knowledge of the Green-Schwarz action. Our starting point is to introduce a sigma-model on the coset space

$$\frac{\mathrm{OSP}(2,2|6)}{\mathrm{SO}(3,1)\times\mathrm{U}(3)}\,. \tag{2.1}$$

Recall that the supergroup[2] $\mathrm{OSP}(2,2|6)$ has a bosonic subgroup $\mathrm{USP}(2,2)\times\mathrm{SO}(6)$; the quotient of the latter over $\mathrm{SO}(3,1)\times\mathrm{U}(3)$ provides a model of the $\mathrm{AdS}_4\times\mathbb{CP}^3$ superspace with $\mathrm{SO}(3,1)\times\mathrm{U}(3)$ playing the role of the local Lorentz group. The superspace obtained in this way is parametrized by 24 (real) fermion degrees of freedom, which is apparently different from both 32 (the gauge unfixed Green-Schwarz superstring) and 16 (the gauge-fixed Green-Schwarz superstring). We then show that the standard kinetic term can be supplemented with the Wess-Zumino term so that the whole action does admit a local fermionic symmetry much analogous to the usual κ-symmetry of the Green-Schwarz superstring. We will show that for generic bosonic configurations the κ-symmetry of the coset model allows one to gauge away precisely 8 fermions, so that the resulting fermionic content match to that of a κ-symmetry fixed version of the Green-Schwarz superstring. This suggests an interpretation of the coset sigma-model with 24 fermions as *the Green-Schwarz superstring with a partial fixing of κ-symmetry* which consists in removing 8 from 32 fermions.

The construction of the Lagrangian for the sigma-model in question is very similar to that for classical superstrings on $\mathrm{AdS}_5\times\mathrm{S}^5$ space [8, 24–26] and it makes use of the $\mathbb{Z}_4$-grading of the $\mathfrak{osp}(2,2|6)$ Lie algebra. We start with recalling the necessary facts about $\mathfrak{osp}(2,2|6)$.

2.2 Superalgebra $\mathfrak{osp}(2,2|6)$ and $\mathbb{Z}_4$-grading

The Lie algebra $\mathfrak{osp}(2,2|6)$ can be realized by 10×10 supermatrices of the form

$$A=\begin{pmatrix} X & \theta \\ \eta & Y\end{pmatrix}\,, \tag{2.2}$$

where X and Y are even (bosonic) 4×4 and 6×6 matrices, respectively. The 4×6 matrix θ and the 6×4 matrix η are odd, i.e. linear in fermionic variables. The matrix A must satisfy the following two conditions

$$A^{\mathrm{st}}\begin{pmatrix} C_4 & 0 \\ 0 & \mathbb{I}_{6\times6}\end{pmatrix}+\begin{pmatrix} C_4 & 0 \\ 0 & \mathbb{I}_{6\times6}\end{pmatrix}A=0 \qquad \Rightarrow \qquad A^{\mathrm{st}}=-\check{C}A\check{C}^{-1}\,, \tag{2.3}$$

$$A^{\dagger}\begin{pmatrix} \Gamma^0 & 0 \\ 0 & -\,\mathbb{I}_{6\times6}\end{pmatrix}+\begin{pmatrix} \Gamma^0 & 0 \\ 0 & -\,\mathbb{I}_{6\times6}\end{pmatrix}A=0 \qquad \Rightarrow \qquad A^{\dagger}=-\check{\Gamma}A\check{\Gamma}^{-1}\,. \tag{2.4}$$

[2]The standard notation for the supergroup is $\mathrm{OSP}(6|4)$. We prefer, however, to use our notation $\mathrm{OSP}(2,2|6)$ to signify that this supergroup is an isometry group of the $\mathrm{AdS}_4\times\mathbb{CP}^3$ superspace. This notation is also in close analogy with $\mathrm{PSU}(2,2|4)$, which is an isometry of the $\mathrm{AdS}_5\times\mathrm{S}^5$ superspace.

Here C_4 is the charge conjugation matrix, and $A^{\rm st}$ denotes the super-transpose matrix

$$A^{\rm st} = \begin{pmatrix} X^t & -\eta^t \\ \theta^t & Y^t \end{pmatrix} . \tag{2.5}$$

We have also introduced four gamma-matrices Γ^μ which satisfy the Clifford algebra of $\mathfrak{so}(3,1)$; their explicit form is given in appendix A. Condition (2.3) singles out $\mathfrak{osp}(4|6)$ with the bosonic subalgebra $\mathfrak{sp}(4,\mathbb{C}) \oplus \mathfrak{so}(6,\mathbb{C})$. Eq. (2.4) defines a real section of $\mathfrak{osp}(4|6)$ which we denote by $\mathfrak{osp}(2,2|6)$.

The charge conjugation matrix can be chosen to be real, skew-symmetric and satisfying $C_4^2 = -\mathbb{I}$, see appendix A for an explicit representation. Conditions (2.3) and (2.4) imply that the matrices X and Y have the following transposition and reality properties

$$X^t = -C_4 X C_4^{-1}\,, \qquad X^* = (i\Gamma^3)X(i\Gamma^3)^{-1} \qquad i\Gamma^3 = \Gamma^0 C_4\,, \tag{2.6}$$

$$Y^t = -Y\,, \qquad Y^* = Y\,, \tag{2.7}$$

while η and θ obey

$$\eta = -\theta^t C_4\,, \qquad \theta^* = i\Gamma^3\theta\,. \tag{2.8}$$

The algebra $\mathfrak{osp}(4|6)$ does not admit an outer automorphism of order four [27]. Thus, we should search for an inner automorphism of order four such that its stationary point would coincide with the subalgebra $\mathfrak{so}(3,1) \times \mathfrak{u}(3)$.

Introduce two 4×4 and 6×6 matrices K_4 and K_6, respectively. We require that $K_4^2 = -\mathbb{I}$ and $K_6^2 = -\mathbb{I}$. In addition, we require $(\Gamma^\mu)^t = K_4\Gamma^\mu K_4^{-1}$ for all gamma-matrices. In what follows it is convenient to make the following choice

$$K_4 = -\Gamma^1\Gamma^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad K_6 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} . \tag{2.9}$$

Define a map

$$\Omega(A) = \begin{pmatrix} K_4 X^t K_4 & K_4\eta^t K_6 \\ -K_6\theta^t K_4 & K_6 Y^t K_6 \end{pmatrix} . \tag{2.10}$$

For any two supermatrices A and B it satisfies the following property

$$\Omega(AB) = -\Omega(B)\Omega(A)$$

and, for this reason, it is an automorphism of $\mathfrak{osp}(4|6)$, i.e.

$$\Omega([A,B]) = -[\Omega(B),\Omega(A)] = [\Omega(A),\Omega(B)]\,.$$

This automorphism is inner. Indeed, using the relations (2.6)–(2.8), we find that

$$\Omega(A)=\begin{pmatrix} K_4C_4 & 0 \\ 0 & -K_6\end{pmatrix}\begin{pmatrix} X & \theta \\ \eta & Y\end{pmatrix}\begin{pmatrix} K_4C_4 & 0 \\ 0 & -K_6\end{pmatrix}^{-1}\equiv \Upsilon A\Upsilon^{-1}\,. \tag{2.11}$$

Since $(K_4C_4)^2=\mathbb{I}$, and $K_6^2=-\mathbb{I}$, the element $\Upsilon\in\mathrm{OSP}(4|6)$ obeys $\Upsilon^4=\mathbb{I}$. In fact, the matrix K_4C_4 coincides with Γ^5 given by $\Gamma^5=-i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$. Further, we note that $\Upsilon^\dagger\check{\Gamma}\Upsilon\check{\Gamma}^{-1}=\mathrm{diag}(-\mathbb{I}_4,\mathbb{I}_6)$. This means that Ω does not preserve the real form $\mathfrak{osp}(2,2|6)$.

The automorphism Ω allows one to endow $\mathcal{A}=\mathfrak{osp}(2,2|6)$ with the structure of a $\mathbb{Z}_4$-graded algebra, i.e., as the vector space $\mathcal{A}$ can be decomposed into a direct sum of four subspaces

$$\mathcal{A}=\mathcal{A}^{(0)}\oplus\mathcal{A}^{(1)}\oplus\mathcal{A}^{(2)}\oplus\mathcal{A}^{(3)} \tag{2.12}$$

such that $[\mathcal{A}^{(k)},\mathcal{A}^{(m)}]\subseteq\mathcal{A}^{(k+m)}$ modulo $\mathbb{Z}_4$. Each subspace $\mathcal{A}^{(k)}$ in eq. (2.12) is an eigenspace of Ω

$$\Omega(\mathcal{A}^{(k)})=i^k\mathcal{A}^{(k)}\,. \tag{2.13}$$

Explicitly, the projection $A^{(k)}$ of a generic element $A\in\mathfrak{osp}(2,2|6)$ on the subspace $\mathcal{A}^{(k)}$ is constructed as follows

$$A^{(k)}=\frac{1}{4}\Big(A+i^{3k}\Omega(A)+i^{2k}\Omega^2(A)+i^k\Omega^3(A)\Big)\,. \tag{2.14}$$

In particular, the stationary subalgebra of Ω is determined by the conditions

$$[\Gamma^5,X]=0\,,\qquad [K_6,Y]=0 \tag{2.15}$$

and it coincides with $\mathfrak{so}(3,1)\times\mathfrak{u}(3)$, see appendix A for details.

The space $A^{(2)}$ is spanned by matrices satisfying the following condition

$$\Omega(A)=\Upsilon A\Upsilon^{-1}=-A\,. \tag{2.16}$$

As is shown in appendix A, any such matrix satisfies the following remarkable identity

$$A^3=\frac{1}{8}\,\mathrm{str}(\Sigma A^2)\,A+\frac{1}{8}\,\mathrm{str}(A^2)\,\Sigma A\,, \tag{2.17}$$

or, equivalently,

$$A^3=\frac{1}{8}(\mathrm{tr}A^2_{\mathrm{AdS}}+\mathrm{tr}A^2_{\mathbb{CP}})\,A+\frac{1}{8}(\mathrm{tr}A^2_{\mathrm{AdS}}-\mathrm{tr}A^2_{\mathbb{CP}})\,\Sigma A\,. \tag{2.18}$$

Here Σ is a diagonal matrix $\Sigma=\Upsilon^2=(\mathbb{I}_4,-\mathbb{I}_6)$. Equation (2.16) boils down to

$$\{X,\Gamma^5\}=0\,,\qquad \{Y,K_6\}=0\,. \tag{2.19}$$

The first equation can be solved as

$$X=x_\mu\Gamma^\mu$$

and it provides a parametrization of the coset space $\mathrm{AdS}_4 = \mathrm{SO}(3,2)/\mathrm{SO}(3,1)$ in terms of four unconstrained variables x_μ. Analogously, a general solution to the second equation in (2.19) gives a parametrization of $\mathbb{CP}^3$

$$Y = y_i T_i,$$

where y_i, $i = 1, \ldots, 6$ are six unconstrained variables and the matrices T_i are described in appendix A.

Finally, for the reader's convenience we present an explicit form of the projections $A^{(1)}$ and $A^{(3)}$ of the matrix A:

$$A^{(1)} = \frac{1}{2}\begin{pmatrix} 0 & \theta - i\Gamma^5\theta K_6 \\ \eta + iK_6\eta\Gamma^5 & 0 \end{pmatrix}, \qquad A^{(3)} = \frac{1}{2}\begin{pmatrix} 0 & \theta + i\Gamma^5\theta K_6 \\ \eta - iK_6\eta\Gamma^5 & 0 \end{pmatrix}.$$

Each of these matrices $A^{(1)}$ and $A^{(3)}$ depend on 12 real fermionic variables.

2.3 The Lagrangian

Let g be an element of the coset (2.1) realized as an embedding in the supergroup $\mathrm{OSP}(2,2|6)$. We use g to build the following current (the one-form)

$$A = -g^{-1}\mathrm{d}g = A^{(0)} + A^{(2)} + A^{(1)} + A^{(3)}. \tag{2.20}$$

The current takes values in the algebra $\mathfrak{osp}(2,2|6)$ and on the r.h.s. of the last formula we exhibited its $\mathbb{Z}_4$-decomposition. By construction A has vanishing curvature:

$$\partial_\alpha A_\beta - \partial_\beta A_\alpha - [A_\alpha, A_\beta] = 0. \tag{2.21}$$

The sigma-model we are looking for is then described by the following action

$$S = -\frac{R^2}{4\pi\alpha'}\int \mathrm{d}\sigma \mathrm{d}\tau\, \mathscr{L}, \tag{2.22}$$

where R is the radius of the AdS space and the Lagrangian density is the sum of the kinetic and the Wess-Zumino terms

$$\mathscr{L} = \gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) + \kappa\epsilon^{\alpha\beta}\mathrm{str}\big(A^{(1)}_\alpha A^{(3)}_\beta\big). \tag{2.23}$$

Here we use the convention $\epsilon^{\tau\sigma} = 1$ and $\gamma^{\alpha\beta} = h^{\alpha\beta}\sqrt{-h}$ is the Weyl-invariant combination of the world-sheet metric $h_{\alpha\beta}$ with $\det\gamma = -1$. The parameter κ in front of the Wess-Zumino term is kept arbitrary for the moment. As we will see shortly, the requirement of κ-symmetry leaves two possibilities $\kappa = \pm 1$. We note that the invariant form defined by means of the supertrace is non-degenerate for the orthosymplectic groups $\mathrm{OSP}(2n|2n+2)$.

Equations of motion derived from this Lagrangian read as

$$\partial_\alpha \Lambda^\alpha - [A_\alpha, \Lambda^\alpha] = 0, \tag{2.24}$$

where we have introduced the combination

$$\Lambda^\alpha = \gamma^{\alpha\beta}A^{(2)}_\beta - \frac{1}{2}\kappa\,\epsilon^{\alpha\beta}\big(A^{(1)}_\beta - A^{(3)}_\beta\big). \tag{2.25}$$

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The equations of motion imply the conservation of the Noether current $J^\alpha = g\Lambda^\alpha g^{-1}$ corresponding to the global $\mathrm{OSP}(2,2|6)$ symmetry of the model: $\partial_\alpha J^\alpha = 0$.

Single equation (2.24) can be projected on various eigenspaces of the $\mathbb{Z}_4$ automorphism. The projection on the subspace $\mathcal{A}^{(0)}$ vanishes. For the projection on $\mathcal{A}^{(2)}$ we get

$$\partial_\alpha(\gamma^{\alpha\beta}A^{(2)}_\beta) - \gamma^{\alpha\beta}[A^{(0)}_\alpha, A^{(2)}_\beta] + \frac{1}{2}\kappa\epsilon^{\alpha\beta}\Big([A^{(1)}_\alpha, A^{(1)}_\beta] - [A^{(3)}_\alpha, A^{(3)}_\beta]\Big) = 0\,, \tag{2.26}$$

while for projections on $\mathcal{A}^{(1,3)}$ one finds

$$\begin{aligned} \gamma^{\alpha\beta}[A^{(3)}_\alpha, A^{(2)}_\beta] + \kappa\epsilon^{\alpha\beta}[A^{(2)}_\alpha, A^{(3)}_\beta] = 0\,,\\ \gamma^{\alpha\beta}[A^{(1)}_\alpha, A^{(2)}_\beta] - \kappa\epsilon^{\alpha\beta}[A^{(2)}_\alpha, A^{(1)}_\beta] = 0\,. \end{aligned} \tag{2.27}$$

In deriving these equations we also used the condition of zero curvature for the connection A_α. Introducing the tensors

$$\mathrm{P}^{\alpha\beta}_\pm = \frac{1}{2}(\gamma^{\alpha\beta} \pm \kappa\epsilon^{\alpha\beta})\,, \tag{2.28}$$

equations (2.27) can be concisely written as

$$\begin{aligned} \mathrm{P}^{\alpha\beta}_-[A^{(2)}_\alpha, A^{(3)}_\beta] = 0\,,\\ \mathrm{P}^{\alpha\beta}_+[A^{(2)}_\alpha, A^{(1)}_\beta] = 0\,. \end{aligned} \tag{2.29}$$

These are equations of motion for fermions. Note that for $\kappa = \pm 1$ the tensors $\mathrm{P}_\pm$ are orthogonal projectors:

$$\mathrm{P}^{\alpha\beta}_+ + \mathrm{P}^{\alpha\beta}_- = \gamma^{\alpha\beta}\,, \qquad \mathrm{P}^{\alpha\delta}_\pm \mathrm{P}_{\pm\delta}^{\ \ \beta} = \mathrm{P}^{\alpha\beta}_\pm\,, \qquad \mathrm{P}^{\alpha\delta}_\pm \mathrm{P}_{\mp\delta}^{\ \ \beta} = 0\,. \tag{2.30}$$

Finally, we also have equations of motion for the world-sheet metric which are the Virasoro constraints:

$$\mathrm{str}(A^{(2)}_\alpha A^{(2)}_\beta) - \frac{1}{2}\gamma_{\alpha\beta}\gamma^{\rho\delta}\mathrm{str}(A^{(2)}_\rho A^{(2)}_\delta) = 0\,. \tag{2.31}$$

We stress that so far the construction of the coset sigma-model does not differ from that for the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [8, 24–26]. The real problem, however, is to show that the above action enjoys a local fermionic symmetry which is capable of gauging away precisely eight fermionic degrees of freedom. This will be the subject of the next section.

3. Local fermionic symmetry

3.1 Deriving κ-symmetry

Kappa-symmetry is a local fermionic symmetry of the Green-Schwarz superstring [22]. It generalizes the local fermionic symmetries first discovered for massive and massless superparticles [28, 29] and its presence is crucial to ensure the space-time supersymmetry of the physical spectrum. In this section we establish κ-symmetry transformations associated with the Lagrangian (2.23).

The action of the global symmetry group $\mathrm{OSP}(2,2|6)$ is realized on a coset element by multiplication from the left. In this respect, κ-symmetry transformations can be understood as the *right local* action of a fermionic element $G = \exp\epsilon$ from $\mathrm{OSP}(2,2|6)$ on a coset representative g [30]:

$$gG(\epsilon) = g' g_c\,, \tag{3.1}$$

where $\epsilon \equiv \epsilon(\tau,\sigma)$ is a local fermionic parameter. Here g_c is a compensating element from $\mathrm{SO}(3,1)\times\mathrm{U}(3)$. The fundamental difference with the case of global symmetry is that for arbitrary ϵ the action is not invariant under the transformation (3.1). Below we find the conditions on ϵ which guarantee the invariance of the action.

First we note that under the local multiplication from the right the connection A transforms as follows

$$\delta_\epsilon A = -\mathrm{d}\epsilon + [A,\epsilon]\,. \tag{3.2}$$

The $\mathbb{Z}_4$-decomposition of this equation gives

$$\begin{aligned}
\delta_\epsilon A^{(1)} &= -\mathrm{d}\epsilon^{(1)} + [A^{(0)},\epsilon^{(1)}] + [A^{(2)},\epsilon^{(3)}]\,,\\
\delta_\epsilon A^{(3)} &= -\mathrm{d}\epsilon^{(3)} + [A^{(0)},\epsilon^{(3)}] + [A^{(2)},\epsilon^{(1)}]\,,\\
\delta_\epsilon A^{(2)} &= [A^{(1)},\epsilon^{(1)}] + [A^{(3)},\epsilon^{(3)}]\,,
\end{aligned} \tag{3.3}$$

where we have assumed that $\epsilon = \epsilon^{(1)} + \epsilon^{(3)}$. Using these formulae we find for the variation of the Lagrangian density

$$\begin{aligned}
\delta_\epsilon \mathscr{L} &= \delta\gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) - 2\gamma^{\alpha\beta}\mathrm{str}\big([A^{(1)}_\alpha, A^{(2)}_\beta]\epsilon^{(1)} + [A^{(3)}_\alpha, A^{(2)}_\beta]\epsilon^{(3)}\big)\\
&\quad + \kappa\epsilon^{\alpha\beta}\mathrm{str}\Big(\partial_\alpha A^{(3)}_\beta\epsilon^{(1)} - \partial_\alpha A^{(1)}_\beta\epsilon^{(3)} + [A^{(0)}_\alpha,\epsilon^{(1)}]A^{(3)}_\beta + [A^{(2)}_\alpha,\epsilon^{(3)}]A^{(3)}_\beta\\
&\qquad + A^{(1)}_\alpha[A^{(0)}_\beta,\epsilon^{(3)}] + A^{(1)}_\alpha[A^{(2)}_\beta,\epsilon^{(1)}]\Big)\,.
\end{aligned} \tag{3.4}$$

Here we used integration by parts to remove the derivatives of ϵ. The variation of the world-sheet metric is left unspecified. Now we note that the zero curvature condition eq. (2.21) implies

$$\begin{aligned}
\epsilon^{\alpha\beta}\partial_\alpha A^{(1)}_\beta &= \epsilon^{\alpha\beta}[A^{(0)}_\alpha, A^{(1)}_\beta] + \epsilon^{\alpha\beta}[A^{(2)}_\alpha, A^{(3)}_\beta]\,,\\
\epsilon^{\alpha\beta}\partial_\alpha A^{(3)}_\beta &= \epsilon^{\alpha\beta}[A^{(0)}_\alpha, A^{(3)}_\beta] + \epsilon^{\alpha\beta}[A^{(1)}_\alpha, A^{(2)}_\beta]\,.
\end{aligned}$$

Taking this into account, we obtain

$$\delta_\epsilon\mathscr{L} = \delta\gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) - 4\,\mathrm{str}\big(\mathrm{P}^{\alpha\beta}_+[A^{(1)}_\beta, A^{(2)}_\alpha]\epsilon^{(1)} + \mathrm{P}^{\alpha\beta}_-[A^{(3)}_\beta, A^{(2)}_\alpha]\epsilon^{(3)}\big)\,.$$

According to this formula, the variation of the Lagrangian trivially vanishes for field configurations which solve equations of motion (2.29) and the Virasoro constraints (2.31). In particular, the variation of the first term is zero due to the identity $\gamma_{\alpha\beta}\delta\gamma^{\alpha\beta} = 0$ which follows from the condition $\det\gamma = -1$. Of course, under κ-symmetry transformations the action should remain invariant *without* using equations of motion.

In what follows we assume that $\kappa = \pm 1$. For any vector V^α we introduce the projections $V^\alpha_\pm$:

$$V^\alpha_\pm = \mathrm{P}^{\alpha\beta}_\pm V_\beta$$

so that the variation of the Lagrangian acquires the form

$$\delta_\epsilon \mathscr{L} = \delta\gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) - 4\,\mathrm{str}\big([A^{(1),\alpha}_+, A^{(2)}_{\alpha,-}]\epsilon^{(1)} + [A^{(3),\alpha}_-, A^{(2)}_{\alpha,+}]\epsilon^{(3)}\big)\,.$$

We further note that from the condition $\mathrm{P}^{\alpha\beta}_\pm A_{\beta,\mp} = 0$ the components $A_{\tau,\pm}$ and $A_{\sigma,\pm}$ are proportional

$$A_{\tau,\pm} = -\frac{\gamma^{\tau\sigma} \mp \kappa}{\gamma^{\tau\tau}} A_{\sigma,\pm}\,. \tag{3.5}$$

To simplify our further treatment, we put for the moment $\epsilon^{(3)} = 0$.

The crucial point of our construction is the following ansatz for the κ-symmetry parameter $\epsilon^{(1)}$

$$\epsilon^{(1)} = A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}\kappa^{\alpha\beta}_{++} + \kappa^{\alpha\beta}_{++}A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-} + A^{(2)}_{\alpha,-}\kappa^{\alpha\beta}_{++}A^{(2)}_{\beta,-} - \frac{1}{8}\mathrm{str}(\Sigma A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-})\kappa^{\alpha\beta}_{++}\,, \tag{3.6}$$

where $\kappa^{\alpha\beta}_{++}$ is the κ-symmetry parameter which is assumed to be independent on the dynamical fields of the model. Obviously, $\kappa^{\alpha\beta}_{++}$ must be an element of $\mathfrak{osp}(2,2|6)$. Since on the product of any three supermatrices A, B and C the automorphism Ω acts as $\Omega(ABC) = \Omega(C)\Omega(B)\Omega(A)$, we see that $\epsilon^{(1)} \in \mathcal{A}^{(1)}$ provided $\kappa^{\alpha\beta}_{++}$ is also an element of degree one: $\kappa^{\alpha\beta}_{++} \in \mathcal{A}^{(1)}$.

Consider now the commutator

$$\begin{aligned}
[A^{(2)}_{\alpha,-}, \epsilon^{(1)}] = {} & A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}A^{(2)}_{\delta,-}\kappa^{\beta\delta}_{++} + A^{(2)}_{\alpha,-}\kappa^{\beta\delta}_{++}A^{(2)}_{\beta,-}A^{(2)}_{\delta,-} + A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}\kappa^{\beta\delta}_{++}A^{(2)}_{\delta,-} \\
& - A^{(2)}_{\beta,-}A^{(2)}_{\delta,-}\kappa^{\beta\delta}_{++}A^{(2)}_{\alpha,-} - \kappa^{\beta\delta}_{++}A^{(2)}_{\beta,-}A^{(2)}_{\delta,-}A^{(2)}_{\alpha,-} - A^{(2)}_{\beta,-}\kappa^{\beta\delta}_{++}A^{(2)}_{\delta,-}A^{(2)}_{\alpha,-} \\
& - \frac{1}{8}\mathrm{str}(\Sigma A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})A^{(2)}_{\alpha,-}\kappa^{\beta\delta}_{++} + \frac{1}{8}\mathrm{str}(\Sigma A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})\kappa^{\beta\delta}_{++}A^{(2)}_{\alpha,-}\,.
\end{aligned} \tag{3.7}$$

Here we have to deal with tensorial structures

$$A^{(2)}_{\alpha,-}\ldots A^{(2)}_{\beta,-}\ldots A^{(2)}_{\delta,-}\,,$$

where dots indicate insertions of other supermatrices, e.g., κ_{++}. Since $A^{(2)}_{\alpha,-}$ is an (anti-)self-dual form, the tensors above are totally symmetric in indices α, β, δ and have, in fact, a single non-trivial entry, all the other entries being proportional to it. Thus, most of the terms in eq. (3.7) are cancelled out and we are left with

$$[A^{(2)}_{\alpha,-}, \epsilon^{(1)}] = \Big[A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}A^{(2)}_{\delta,-} - \frac{1}{8}\mathrm{str}(\Sigma A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})A^{(2)}_{\alpha,-}, \kappa^{\beta\delta}_{++}\Big]\,. \tag{3.8}$$

Now we invoke the identity (2.18) satisfied by any element $A \in \mathcal{A}^{(2)}$, according to which

$$A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}A^{(2)}_{\delta,-} - \frac{1}{8}\mathrm{str}(\Sigma A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})A^{(2)}_{\alpha,-} = \frac{1}{8}\mathrm{str}(A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})\Sigma A^{(2)}_{\alpha,-}\,.$$

Thus, we have found that

$$[A^{(2)}_{\alpha,-},\epsilon^{(1)}] = \frac{1}{8}\,\mathrm{str}(A^{(2)}_{\beta,-}A^{(2)}_{\delta,-})[\Sigma A^{(2)}_{\alpha,-},\kappa^{\beta\delta}_{++}]\,. \tag{3.9}$$

Now we see that the κ-symmetry variation of the action

$$\delta_\epsilon \mathscr{L} = \delta\gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) - 4\,\mathrm{str}\Big([A^{(1),\alpha}_+, A^{(2)}_{\alpha,-}]\epsilon^{(1)}\Big) \tag{3.10}$$

implies then the following transformation law for the two-dimensional metric

$$\delta\gamma^{\alpha\beta} = \frac{1}{2}\,\mathrm{str}\Big(\Sigma A^{(2)}_{\delta,-}[\kappa^{\alpha\beta}_{++}, A^{(1),\delta}_+]\Big)\,. \tag{3.11}$$

Notice that the condition $\gamma_{\alpha\beta}\delta\gamma^{\alpha\beta}$ is automatically obeyed because

$$\gamma_{\alpha\beta}\delta\gamma^{\alpha\beta} = \gamma^{\alpha\beta}P^+_{\alpha\delta}P^+_{\beta\eta}\kappa^{\delta\eta} = 0\,.$$

Using the fact that the matrix Σ anti-commutes with any fermionic matrix, we can rewrite the κ-variation of the metric as

$$\delta\gamma^{\alpha\beta} = \frac{1}{2}\,\mathrm{str}\Big(\Sigma\kappa^{\alpha\beta}_{++}\{A^{(1),\delta}_+, A^{(2)}_{\delta,-}\}\Big)\,. \tag{3.12}$$

We see that in a certain sense the variation occurs in the direction orthogonal to the fermionic equations of motion which are $[A^{(1),\delta}_+, A^{(2)}_{\delta,-}] = 0$.

It is obvious that the treatment above can be repeated for the variation involving $\epsilon^{(3)}$, so that a complete variation of the metric under κ-symmetry will be of the form

$$\delta\gamma^{\alpha\beta} = \frac{1}{2}\,\mathrm{str}\Big(\Sigma A^{(2)}_{\delta,-}[\kappa^{\alpha\beta}_{++}, A^{(1),\delta}_+]\Big) + \frac{1}{2}\,\mathrm{str}\Big(\Sigma A^{(2)}_{\delta,+}[\varkappa^{\alpha\beta}_{--}, A^{(3),\delta}_-]\Big)\,, \tag{3.13}$$

where $\varkappa^{\alpha\beta}_{--}\subset\mathcal{A}^{(3)}$ is another independent κ-symmetry parameter.

We would like to point out that in our derivation of κ-symmetry we used the fact that $\mathrm{P}^{\alpha\beta}_{\pm}$ are orthogonal projectors and, therefore, realization of the κ-symmetry requires the parameter κ in the Lagrangian to be equal to ± 1.

3.2 Rank of κ-symmetry transformations on-shell

The next important question is to understand how many fermionic degrees of freedom can be gauged away on-shell by means of κ-symmetry. To this end one can make use of the light-cone gauge. Generically, the light-cone coordinates $X^\pm$ are introduced by making linear combinations of one field corresponding to the time direction from AdS_4 and one field from $\mathbb{CP}^3$. Without loss of generality we can assume that the transversal fluctuation are all suppressed and the corresponding element $A^{(2)}$ has the form

$$A^{(2)} = \begin{pmatrix} ix\Gamma^0 & 0 \\ 0 & yT_6 \end{pmatrix}\,. \tag{3.14}$$

Indeed, the matrix Γ^0 corresponds to the time direction in AdS_4 and any element from the tangent space to $\mathbb{CP}^3$ can be brought to T_6 by means of an $\mathfrak{so}(6)$ transformation. The

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Virasoro constraint $\mathrm{str}(A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-})=0$ then demands that $x^2=y^2$, i.e. $x=\pm y$. Picking up, e.g., the first solution $x=y$, we then compute the element $\epsilon^{(1)}$ assuming a generic parameter κ, which depends on 12 independent fermionic variables.[3] First, we see that $\mathrm{str}(\Sigma A^{(2)}A^{(2)})=-8x^2$. Second, plugging eq. (3.14) into eq. (3.6), we obtain

$$\epsilon^{(1)}=x^2\begin{pmatrix}0 & \varepsilon\\ -\varepsilon^t C_4 & 0\end{pmatrix}, \tag{3.15}$$

where ε is the following matrix

$$\varepsilon=\begin{pmatrix}0 & 0 & i(ik_{13}-k_{16}) & i(ik_{14}-k_{15}) & ik_{14}-k_{15} & ik_{13}-k_{16}\\ 0 & 0 & i(ik_{23}-k_{26}) & i(ik_{24}-k_{26}) & ik_{24}-k_{25} & ik_{23}-k_{26}\\ 0 & 0 & -i(-ik_{33}-k_{36}) & -i(-ik_{34}-k_{35}) & -ik_{34}-k_{35} & -ik_{33}-k_{36}\\ 0 & 0 & -i(-ik_{43}-k_{46}) & -i(-ik_{44}-k_{45}) & -ik_{44}-k_{45} & -ik_{43}-k_{46}\end{pmatrix}$$

and $\kappa_{ij}\equiv\kappa_{++,ij}$ are the entries of the matrix κ_{++}. As we see, the matrix ε depends on 8 independent complex fermionic parameters (e.g. the last two columns). The reality condition (2.8) for ε reduces this number by half. Finally, $\epsilon^{(1)}$ must belong to the component $\mathcal{A}^{(1)}$ which further reduce the number of fermions by half. As the result, $\epsilon^{(1)}$ depends on four real fermionic parameters. A similar analysis shows that $\epsilon^{(3)}$ will also depend on four real fermions. Thus, in total $\epsilon^{(1)}$ and $\epsilon^{(3)}$ depend on 8 real fermions and these are those degrees of freedom which can be gauged away by κ-symmetry. The gauge-fixed coset model will therefore involve 16 physical fermions only.

It should be noted that the considerations above are applicable to a generic case, where string motion occurs in both AdS_4 and $\mathbb{CP}^3$ spaces. There is however a singular situation, when string moves in the AdS space only (e.g. the string spinning in AdS_3 [31]). One can show that for this case the transformation (3.6) vanishes, although the fermionic equations remain degenerate and only 12 of them (out of 24) are independent. This suggests that realization of κ-symmetry changes in this singular situation and κ-symmetry becomes capable of gauging away 12 from 24 fermions. A singular nature of the corresponding bosonic background shows up in the fact that as soon as fluctuations along $\mathbb{CP}^3$ directions are switched on, the rank of κ-symmetry gets reduced to 8. As the result, singular backgrounds cannot be quantized semi-classically within the coset sigma-model. This picture is rather different from that for conventional type IIA or IIB superstrings. There κ-symmetry can always remove half of fermionic degrees of freedom. Which half however, does depend on a chosen bosonic background. As was already explained above, we would like to treat our coset model as the one which originates from the type IIA superstring on $\mathrm{AdS}_4\times\mathbb{CP}^3$ upon a partial κ-symmetry fixing. The trouble with singular (AdS) backgrounds we observe here could be, therefore, due to their incompatibility with the κ-symmetry gauge choice which reduces type IIA superstring on $\mathrm{AdS}_4\times\mathbb{CP}^3$ to our coset model. It would be important to further clarify this issue.

[3] The matrix κ_{++} depends on 12 fermionic variables only, because it is an element of $\mathcal{A}^{(1)}$.

4. Integrability

Since there is no difference in construction of the Lagrangian in comparison to the case of $\mathrm{AdS}_5\times\mathrm{S}^5$, the Lax connection found in [13] for superstrings on $\mathrm{AdS}_5\times\mathrm{S}^5$ is applicable to our model as well and, therefore, we can straightforwardly conclude its kinematical integrability. The main result of this section consists in showing that under κ-symmetry variations found in the previous section the Lax connection undergoes a gauge transformation on-shell. This provides another non-trivial check that κ-symmetry transforms solution of the equations of motion into solutions.

4.1 Lax connection

The Lax representation of the superstring equations of motion on $\mathrm{AdS}_5\times\mathrm{S}^5$ has been found in [13]. The corresponding two-dimensional Lax connection L_α has the following structure

$$L_\alpha = \ell_0 A^{(0)}_\alpha + \ell_1 A^{(2)}_\alpha + \ell_2\gamma_{\alpha\beta}\epsilon^{\beta\rho}A^{(2)}_\rho + \ell_3 A^{(1)}_\alpha + \ell_4 A^{(3)}_\alpha\,, \tag{4.1}$$

where ℓ_i are some constants.

The connection L is required to have zero curvature as a consequence of the dynamical equations and the flatness of A_α. This requirement allows one to determine the constants ℓ_i. For the reader's convenience below we summarize the result for ℓ_i.

All the parameters ℓ_i are determined in terms of ℓ_1:

$$\ell_3^2 = \ell_1 \pm \sqrt{\ell_1^2-1}\,, \qquad \ell_4^2 = \ell_1 \mp \sqrt{\ell_1^2-1}\,, \qquad \ell_2 = \pm\sqrt{\ell_1^2-1}\,, \quad \ell_0 = 1\,. \tag{4.2}$$

The signs in these formulae correlate with the corresponding sign of κ which is also required to satisfy the condition $\kappa^2=1$. It is convenient to describe all the coefficients in terms of uniformizing spectral parameter z. We parametrize

$$\ell_1 = \frac{1+z^2}{1-z^2}\,.$$

For the remaining coefficients ℓ_i the complete set of solutions reads as follows

$$\ell_2 = s_1\frac{2z}{1-z^2}\,, \qquad \ell_3 = s_2\frac{1+s_2s_3z}{\sqrt{1-z^2}}\,, \qquad \ell_4 = s_2\frac{1-s_2s_3z}{\sqrt{1-z^2}}\,, \tag{4.3}$$

Here $s_2^2=s_3^2=1$ and $s_1s_2s_3=-\operatorname{sign}\kappa$. Thus, for every choice of κ we have four different solutions for ℓ_i specified by the choice of $s_2=\pm1$ and $s_3=\pm1$. The spectral parameter z takes values in the complex plane and, for this reason, the Lax connection takes values in the complexified algebra $\mathfrak{osp}(4|6)$.

Finally, we point out how the grading map Ω acts on the Lax connection L_α. Since Ω is the automorphism of $\mathfrak{osp}(2,2|6)$ the curvature of $\Omega(L_\alpha)$ also vanishes. It can be easily checked that $\Omega(L_\alpha)$ is related to L_α by a certain diffeomorphism of the spectral parameter, namely,

$$\Omega(L_\alpha(z)) = \Upsilon L_\alpha(z)\Upsilon^{-1} = L_\alpha(1/z)\,.$$

In summary, equations of motion admit the same zero-curvature representation as for superstring on $\mathrm{AdS}_5\times\mathrm{S}^5$ which ensures the kinematical integrability of our coset model. Inclusion of the Wess-Zumino term is allowed by integrability only for $\kappa=\pm1$, i.e. only for those values of κ for which the model has the local fermionic symmetry.

4.2 κ-variations of the Lax connection

In this section we analyse the relationship between the Lax connection and κ-symmetry. By using the formulae (3.3) which describe how the $\mathbb{Z}_4$-components of $A = -g^{-1}\mathrm{d}g$ transform under the κ-symmetry, it is straightforward to find the κ-symmetry variation of the Lax connection

$$\delta L_\alpha = [L_\alpha, \Lambda] - \partial_\alpha \Lambda + \ell_2\ell_3\epsilon_{\alpha\beta}[A_-^{(2),\beta}, \epsilon^{(1)}] + \ell_2\epsilon_{\alpha\beta}\Big(2[A_+^{(1),\beta}, \epsilon^{(1)}] + \delta\gamma^{\beta\delta}A_\delta^{(2)}\Big)\,, \quad (4.4)$$

where $\Lambda = \ell_3\epsilon^{(1)}$. The last two terms proportional to $\ell_2\ell_3$ and ℓ_2 would destroy the zero curvature condition for the κ-transformed connection unless they separately vanish. Concerning the first term, as we have shown in the previous section,

$$[A_{\alpha,-}^{(2)}, \epsilon^{(1)}] = \frac{1}{8}\mathrm{str}(A_{\beta,-}^{(2)}A_{\delta,-}^{(2)})[\Sigma A_{\alpha,-}^{(2)}, \kappa_{++}^{\beta\delta}]\,, \quad (4.5)$$

so that this term vanishes due to the Virasoro constraints:

$$\mathrm{str}(A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)}) = 0\,.$$

As to the second term, by using equations of motion for fermions the relevant commutator can be written as follows

$$\begin{aligned}[A_+^{(1),\beta}, \epsilon^{(1)}] = {}& A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)}[A_+^{(1),\beta}, \kappa_{++}^{\alpha\beta}] + [A_+^{(1),\beta}, \kappa_{++}^{\alpha\beta}]A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)} \\ &+ A_{\alpha,-}^{(2)}[A_+^{(1),\beta}, \kappa_{++}^{\alpha\beta}]A_{\beta,-}^{(2)} - \frac{1}{8}\,\mathrm{str}(\Sigma A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)})[A_+^{(1),\beta}, \kappa_{++}^{\alpha\beta}]\,. \end{aligned} \quad (4.6)$$

We assume a parametrization

$$A^{(2)} = \begin{pmatrix} y_\mu\Gamma^\mu & 0 \\ 0 & \bar{y}_iT_i \end{pmatrix}\,, \qquad [A_+^{(1),\beta}, \kappa_{++}^{\alpha\beta}] = \begin{pmatrix} u_\mu\Gamma^\mu & 0 \\ 0 & \bar{u}_iT_i \end{pmatrix} \quad (4.7)$$

and, therefore, $\frac{1}{8}\,\mathrm{str}(\Sigma A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)}) = \frac{1}{2}(y^2 - \bar{y}^2)$, while the Virasoro constraint is

$$\mathrm{str}(A_{\alpha,-}^{(2)}A_{\beta,-}^{(2)}) = 4(y^2 + \bar{y}^2) = 0 \quad \Longrightarrow \quad y^2 = -\bar{y}^2\,. \quad (4.8)$$

With this parametrization at hand, the r.h.s of eq. (4.6) boils down to the following matrix expression

$$\begin{aligned}[A_+^{(1),\beta}, \epsilon^{(1)}] = {}& \begin{pmatrix} y_\mu y_\nu u_\rho\Big(\Gamma^\mu\Gamma^\nu\Gamma^\rho + \Gamma^\mu\Gamma^\rho\Gamma^\nu + \Gamma^\rho\Gamma^\mu\Gamma^\nu\Big) & 0 \\ 0 & \bar{y}_i\bar{y}_j\bar{u}_k\Big(T_iT_jT_k + T_iT_kT_j + T_kT_iT_j\Big) \end{pmatrix} \\ &- \frac{1}{2}(y^2 - \bar{y}^2)\begin{pmatrix} u_\mu\Gamma^\mu & 0 \\ 0 & \bar{u}_iT_i \end{pmatrix}\,. \end{aligned}$$

The Clifford algebra for the gamma-matrices together with the permutation properties for T_i's allows one to rewrite the above formula as

$$[A_+^{(1),\beta}, \epsilon^{(1)}] = \begin{pmatrix} y^2u_\mu\Gamma^\mu + 2(yu)y_\mu\Gamma^\mu & 0 \\ 0 & -\bar{y}^2\bar{u}_iT_i - 2(\bar{y}\bar{u})\bar{y}_iT_i \end{pmatrix} - \frac{1}{2}(y^2 - \bar{y}^2)\begin{pmatrix} u_\mu\Gamma^\mu & 0 \\ 0 & \bar{u}_iT_i \end{pmatrix}\,,$$

where we defined $(yu) \equiv y_\mu u_\nu \eta^{\mu\nu}$ and $(\bar{y}\bar{u}) \equiv \bar{y}_i \bar{u}_i$. Taking into account the Virasoro constraints, the last expression simplifies to

$$[A_+^{(1),\beta}, \epsilon^{(1)}] = \begin{pmatrix} 2(yu)\, y_\mu \Gamma^\mu & 0 \\ 0 & -2(\bar{y}\bar{u})\, \bar{y}_i T_i \end{pmatrix} . \tag{4.9}$$

We also notice that due to the fermion equations of motion the products (yu) and $(\bar{y}\bar{u})$ are not independent. Indeed, on-shell we have

$$0 = \mathrm{str}([A_+^{(1),\alpha}, \epsilon^{(1)}] A_{\alpha,-}^{(2)}) = 8(y^2(yu) - \bar{y}^2(\bar{y}\bar{u})) . \tag{4.10}$$

The Virasoro constraints (4.8) then imply that

$$(yu) = -(\bar{y}\bar{u}) . \tag{4.11}$$

On the other hand,

$$\delta\gamma^{\beta\delta} A_\delta^{(2)} = -2((yu) - (\bar{y}\bar{u})) \begin{pmatrix} y_\mu \Gamma^\mu & 0 \\ 0 & \bar{y}_i T_i \end{pmatrix} = -4(yu) \begin{pmatrix} y_\mu \Gamma^\mu & 0 \\ 0 & \bar{y}_i T_i \end{pmatrix} . \tag{4.12}$$

Thus,

$$2[A_+^{(1),\beta}, \epsilon^{(1)}] + \delta\gamma^{\beta\delta} A_\delta^{(2)} = 0 , \tag{4.13}$$

i.e. a κ-symmetry variation of the Lax connection is a gauge transformation on-shell.

5. Plane-wave limit

In this section we discuss the perturbative expansion of the string sigma model action up to the quadratic order in the bosonic and fermionic fields around a point-like string solution describing a massless particle moving in $\mathbb{CP}^3$ along a null geodesic given by the equations $w_1 = w_2 = 0$, $w_3 = e^{i\phi}$. The reader should consult appendix B for notations and parametrizations of $\mathbb{CP}^3$ used in the paper. The expansion corresponds to taking a Penrose or plane-wave limit of the background $\mathrm{AdS}_4 \times \mathbb{CP}^3$ geometry which was recently discussed in [11, 12]. The resulting type IIA string theory pp-wave background has 24 supersymmetries, and the corresponding light-cone gauge Green-Schwarz action describes 8 massive bosons and 8 massive fermions, and was constructed in [9, 10]. We use the sigma model action (2.22) to compute the quadratic action, then we impose a certain κ-symmetry gauge condition and show that the light-cone gauge action coincides with the one in [9, 10]. We consider this computation as a first nontrivial check of our coset sigma model action for superstrings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$.

To find a reasonably good expansion around the geodesics, it is convenient to use the homogeneous coordinates z_i of $\mathbb{CP}^3$. Then one can see that the parametrization of z_i which leads to a simple bosonic quadratic action describing massive excitations can be chosen as follows

$$z_4 = e^{-i\phi/2} , \quad z_3 = (1 - x_4) e^{i\phi/2} , \quad z_1 = \frac{1}{\sqrt{2}} y_1 , \quad z_2 = \frac{1}{\sqrt{2}} y_2 ,$$

where the angle ϕ parametrizes the geodesics, and the complex coordinates y_1, y_2 and the real coordinate x_4 denote the five physical fluctuations in $\mathbb{CP}^3$. In terms of the inhomogeneous coordinates w_i the parametrization has the form

$$w_3 = (1 - x_4)e^{i\phi}\,, \quad w_1 = \frac{1}{\sqrt{2}} y_1 e^{i\phi/2}\,, \quad w_2 = \frac{1}{\sqrt{2}} y_2 e^{i\phi/2}\,,$$

and all the coordinates w_i depend on ϕ.

Then, expanding the $\mathbb{CP}^3$ metric (B.11) in powers of x_4, y_1, y_2, one finds

$$4\, ds^2_{\mathbb{CP}^3} = d\phi^2 \left(1 - x_4^2 - \frac{1}{4}\bar{y}_r y_r\right) + dx_4^2 + d\bar{y}_r dy_r + \cdots\,, \quad r = 1, 2\,.$$

This formula should be combined with the standard expansion of the AdS_4 metric, see e.g. [14] for a convenient parametrization of AdS_d

$$ds^2_{\mathrm{AdS}_4} = -dt^2(1 + x_i^2) + dx_i^2 + \cdots\,, \quad i = 1, 2, 3\,,$$

where t is the global time coordinate, and x_i are three physical fluctuations in AdS_4.

Thus, the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background metric admits the following expansion

$$ds^2_{\mathrm{AdS}_4 \times \mathbb{CP}^3} = -dt^2(1 + x_i^2) + dx_i^2 + d\phi^2 \left(1 - x_4^2 - \frac{1}{4}\bar{y}_r y_r\right) + dx_4^2 + d\bar{y}_r dy_r + \cdots\,. \tag{5.1}$$

It is clear now that plugging in the point-like string solution with $t = \tau\,,\ \phi = \tau$ in the corresponding string Lagrangian (2.23) one gets four massive fields of mass 1/2 and four fields of mass 1. Note that the field x_4 from $\mathbb{CP}^3$ joins the three fields from AdS_4. It is unclear at the moment if it is a consequence of the supersymmetry or an artifact of the plane-wave expansion and sigma-model loop corrections would result in a mass splitting.

To find the quadratic fermion action in the background, we need to know the coset representative corresponding to the point-like string solution. Since for the solution $w_1 = w_2 = 0\,,\ w_3 = e^{i\phi}$, it is given by

$$g_B = \begin{pmatrix} g_{\mathrm{AdS}} & 0 \\ 0 & g_{\mathbb{CP}} \end{pmatrix}\,, \quad g_{\mathrm{AdS}} = e^{it\Gamma^0/2}\,, \tag{5.2}$$

$$g_{\mathbb{CP}} = I + \frac{e^{i\phi}\mathcal{T}_3 + e^{-i\phi}\overline{\mathcal{T}}_3}{\sqrt{2}} + \left(1 - \frac{1}{\sqrt{2}}\right)(\mathcal{T}_3\overline{\mathcal{T}}_3 + \overline{\mathcal{T}}_3\mathcal{T}_3)\,, \tag{5.3}$$

where we use (B.8), and take into account that the time direction corresponds to Γ^0.

Then we build up the group element in the form

$$g = g(\chi)\, g_B \tag{5.4}$$

and compute the quadratic part of the fermion Lagrangian (2.23). In the last formula $g(\chi) = \exp\chi$, where χ is a generic odd element of $\mathfrak{osp}(2,2|6)$.

One can check that the coset representative $g_{\mathbb{CP}}$ (5.3) does not correspond to any one-parameter subgroup of SO(6) because the tangent element to $g_{\mathbb{CP}}$ has an explicit dependence on ϕ. For this reason it seems easier to parametrize the coset manifolds AdS_4 and $\mathbb{CP}^3$ by the following group elements

$$G = \mathrm{diag}(G_{\mathrm{AdS}},\ G_{\mathbb{CP}})\,, \quad G_{\mathrm{AdS}} = g_{\mathrm{AdS}}\, K_4\, g^t_{\mathrm{AdS}}\,, \quad G_{\mathbb{CP}} = g_{\mathbb{CP}}\, K_6\, g^t_{\mathbb{CP}}\,. \tag{5.5}$$

Indeed, as was shown in [32], the sigma model string Lagrangian of the form (2.23) can be rewritten in terms of these elements as follows

$$\begin{aligned}\mathscr{L} = \frac{1}{4}\,\mathrm{str}\Big[&\gamma^{\alpha\beta}(\mathrm{B}_\alpha + G\mathrm{B}^t_\alpha G^{-1} + \partial_\alpha G G^{-1})(\mathrm{B}_\beta + G\mathrm{B}^t_\beta G^{-1} + \partial_\beta G G^{-1}) \\ &+ 2i\kappa\epsilon^{\alpha\beta}\mathrm{F}_\alpha G \mathrm{F}^{\mathrm{st}}_\beta G^{-1}\Big]\,,\end{aligned} \tag{5.6}$$

where F and B are odd and even superalgebra elements made of fermions only

$$g^{-1}(\chi)dg(\chi) = \mathrm{F} + \mathrm{B}\,, \quad \mathrm{F} = d\chi + \cdots\,, \quad \mathrm{B} = \frac{1}{2}d\chi\chi - \frac{1}{2}\chi d\chi + \cdots\,. \tag{5.7}$$

The coset group elements are skew-symmetric matrices $G^t = -G$, and, therefore, AdS_4 can be identified with the intersection of 4×4 skew-symmetric matrices with USP(2,2) ones, and $\mathbb{CP}^3$ with the intersection of 6×6 skew-symmetric and orthogonal matrices. The parametrizations (5.4) of the supergroup elements and (5.5) of the coset manifolds are distinguished because the bosonic subgroup of $\mathrm{OSP}(2,2|6)$ acts on the coset representatives $g(\chi)$ and G by the usual matrix conjugation [33].

The string Lagrangian (5.6) can be further simplified by taking into account that

$$\mathrm{str}\left(G\mathrm{B}^t_\alpha G^{-1}\partial_\beta G G^{-1}\right) = \mathrm{str}\left(\mathrm{B}_\alpha \partial_\beta G G^{-1}\right)\,,$$

and, therefore, we can bring (5.6) to the following simple form

$$\begin{aligned}\mathscr{L} = \frac{1}{4}\mathrm{str}\Big[&\gamma^{\alpha\beta}(\partial_\alpha G G^{-1}\partial_\beta G G^{-1} + 4\mathrm{B}_\alpha\partial_\beta G G^{-1} + 2\mathrm{B}_\alpha\mathrm{B}_\beta + 2\mathrm{B}_\alpha G\mathrm{B}^t_\beta G^{-1}) \\ &+2i\kappa\,\epsilon^{\alpha\beta}\mathrm{F}_\alpha G\mathrm{F}^{\mathrm{st}}_\beta G^{-1}\Big]\,.\end{aligned} \tag{5.8}$$

The first term in the expression gives the bosonic part of the string Lagrangian determined by the background metric. According to formula (5.1), the corresponding action expanded around the particle trajectory $t=\tau\,,\ \phi=\tau$ takes the following form

$$S^{(2)}_B = -\frac{R^2}{4\pi\alpha'}\int_0^{\mathcal{J}} \mathrm{d}\sigma\mathrm{d}\tau\left(\partial^\alpha x_k\partial_\alpha x_k - x_k^2 + \partial^\alpha\bar{y}_r\partial_\alpha y_r - \frac{1}{4}\bar{y}_r y_r\right), \tag{5.9}$$

where $k=1,2,3,4$, the integration limit $\mathcal{J}$ is determined by the charge (or target space-time energy) carried by the particle: $E = J = \frac{R^2}{2\pi\alpha'}\mathcal{J}$, and we dropped the unessential fluctuations in the time and ϕ directions.

The quadratic fermion action in the particle background is given by the sum of the second and fifth terms in (5.8)

$$\mathscr{L}^{(2)}_F = \mathrm{str}\left[\gamma^{\alpha\beta}\mathrm{B}_\alpha\partial_\beta G G^{-1} + \frac{i}{2}\kappa\epsilon^{\alpha\beta}\mathrm{F}_\alpha G\mathrm{F}^{\mathrm{st}}_\beta G^{-1}\right]\,, \tag{5.10}$$

where

$$\mathrm{F}=d\chi\,,\quad \mathrm{B}=\frac{1}{2}d\chi\chi-\frac{1}{2}\chi d\chi\,, \tag{5.11}$$

and G is given by (5.5), (5.2) and (5.3) with $t=\phi=\tau$.

Then, one can show that $G_{\mathbb{CP}}$ with $g_{\mathbb{CP}}$ given by eq. (5.3) admits the following simple representation

$$G_{\mathbb{CP}}=g_{\mathbb{CP}}\,K_6\,g^t_{\mathbb{CP}}=(I+e^{i\phi}\mathcal{T}_3+e^{-i\phi}\overline{\mathcal{T}}_3+\mathcal{T}_3\overline{\mathcal{T}}_3+\overline{\mathcal{T}}_3\mathcal{T}_3)K_6=h(\phi)\,K_6\,h(\phi)^t\,,$$

where

$$h(\phi)=e^{-\phi T_{56}}e^{\frac{\pi}{2}T_{35}}\,,\quad T_{ij}=E_{ij}-E_{ji}\,.$$

Therefore, we can redefine the fermions and bosons as

$$G\to HGH^t\,,\quad \chi\to H\,\chi\,H^{-1}\,,\quad H=\mathrm{diag}(g_{\mathrm{AdS}}(t),\ h(\phi))\,,$$

and remove the explicit dependence of t and ϕ from the Lagrangian leaving only the dependence of their derivatives. One can easily see that the redefinition amounts to the following replacement in the terms F and B of the Lagrangian (5.10)

$$d\chi\to D\chi=d\chi+[dh\,,\chi]\,,$$

where

$$dh=H^{-1}dH=\mathrm{diag}(dh_{\mathrm{AdS}}\,,dh_{\mathbb{CP}})=\mathrm{diag}\left(\frac{i}{2}\Gamma^0dt\,,\ T_{36}\,d\phi\right)\,,$$

and the transformed G is just the constant matrix $G=K=\mathrm{diag}(K_4,K_6)$, and

$$dGG^{-1}=dh-Kdh^tK=\mathrm{diag}(i\Gamma^0dt\,,\ (T_{36}+KT_{36}K)\,d\phi)=\mathrm{diag}(i\Gamma^0dt\,,\ T_6\,d\phi)\,.$$

Taking into account that if all bosonic fluctuations vanish then the world-sheet metric is flat, we conclude that the quadratic fermion Lagrangian (5.10) is equal to

$$\begin{aligned}\mathscr{L}^{(2)}_F&=\mathrm{str}\left[\mathrm{B}_0\partial_0GG^{-1}-i\kappa\mathrm{F}_0K\partial_1\chi^{\mathrm{st}}K^{-1}\right] \qquad (5.12)\\&=\mathrm{str}\Bigg[\frac{1}{2}(\partial_0\chi\chi-\chi\partial_0\chi)(i\Gamma^0+T_6))+i\kappa\left[\frac{i}{2}\Gamma^0+T_{36}\,,\chi\right]K\partial_1\chi^{\mathrm{st}}K\\&\quad+\frac{1}{2}\left(\left(\frac{i}{2}\Gamma^0+T_{36}\right)\chi^2+\chi^2\left(\frac{i}{2}\Gamma^0+T_{36}\right)-2\chi\left(\frac{i}{2}\Gamma^0+T_{36}\right)\chi\right)(i\Gamma^0+T_6)\Bigg]\,,\end{aligned}$$

where we use the obvious embedding of the matrices $\Gamma^0\,,\ T_6\,,\ T_{36}$ into $\mathfrak{osp}(2,2|6)$.

The Lagrangian (5.12) is invariant under κ-symmetry transformations discussed in section 3. The symmetry allows one to impose the gauge-fixing condition

$$\chi\,T_{56}=0\,, \tag{5.13}$$

which implies that the last two columns of θ and last two rows of η vanish leaving only 16 physical fermion degrees of freedom.

Then one can easily check that due to the gauge fixing $\mathrm{str}\,(\partial_0\chi\chi - \chi\partial_0\chi)T_6 = 0$, and $\mathrm{str}\,[T_{36}\,,\chi]K\partial_1\chi^{\mathrm{st}}K = 0$. Hence, the kinetic term in eq. (5.12) becomes non-degenerate and equal to

$$\mathrm{str}\left[\frac{1}{2}(\partial_0\chi\chi - \chi\partial_0\chi)(i\Gamma^0 + T_6))\right] = -i\,\mathrm{tr}\,\eta\Gamma^0\dot{\theta} = i\,\mathrm{tr}\,\theta^t C_4\Gamma^0\dot{\theta} = \mathrm{tr}\,\theta^t\Gamma^3\dot{\theta}\,,$$

and the σ-derivative term is given by

$$\mathrm{str}\left(i\kappa\left[\frac{i}{2}\Gamma^0\,,\chi\right]K\partial_1\chi^{\mathrm{st}}K\right) = \kappa\,\mathrm{tr}\,\theta^t\,\Gamma^0\,K_4\,\theta'\,K_6\,.$$

Computing the mass term, we get

$$\begin{aligned}&\mathrm{str}\left[\frac{1}{2}\left(\left(\frac{i}{2}\Gamma^0 + T_{36}\right)\chi^2 + \chi^2\left(\frac{i}{2}\Gamma^0 + T_{36}\right) - 2\chi\left(\frac{i}{2}\Gamma^0 + T_{36}\right)\chi\right)(i\Gamma^0 + T_6)\right]\\ &= -\frac{1}{2}\,\mathrm{tr}\left[\theta^t C_4\theta(I - \{T_6, T_{36}\})\right].\end{aligned}$$

Finally, introducing a fermion 4 by 4 matrix ϑ made of nonvanishing entries of θ we can write the quadratic Lagrangian in the form (with $\kappa = 1$)

$$\mathscr{L}_F^{(2)} = \mathrm{tr}\left(\vartheta^t\Gamma^3\dot{\vartheta} + \vartheta^t\,\Gamma^0\,K_4\,\vartheta'\,K_4 - \frac{1}{2}\,\vartheta^t C_4\vartheta D_4\right), \tag{5.14}$$

where $D_4 = \mathrm{diag}(1, 1, 3, 1)$ is the restriction of $I - \{T_6, T_{36}\}$ to the first four entries.

Computing the spectrum, one finds that the Lagrangian (5.14) describes eight fermions with frequencies $\omega_p = \sqrt{p^2 + \frac{1}{4}}$, four fermions with frequencies $\omega_p = -\frac{1}{2} + \sqrt{p^2 + 1}$, and four fermions with frequencies $\omega_p = \frac{1}{2} + \sqrt{p^2 + 1}$. It is clear from the spectrum that the fermion Lagrangian (5.14) describes eight fermions of mass 1/2 and eight fermions of mass 1 because the constants $\pm 1/2$ in the last eight frequencies can be removed by a time-dependent redefinition of the corresponding fermions. In fact, the time dependence reflects the fact that some of the fermions are still charged with respect to the $\mathrm{U}(1)$ subgroup that generates the shifts of the angle variable ϕ.

It is easy to guess that the right fermion spectrum, i.e. the one without any constant shifts by $\pm 1/2$ in the frequencies, is obtained from eq. (5.14) by replacing D_4 by the matrix $\mathrm{diag}(1, 1, 2, 2)$ which is the restriction of $I - T_6^2$ to the first four entries. This replacement is just a subtraction of the matrix $\mathrm{diag}(I_2, \sigma_3)$ from D_4, and this suggests to represent the fermion ϑ in the following block form

$$\vartheta = \begin{pmatrix} \vartheta_1 & \zeta_1 \\ \vartheta_2 & \zeta_2 \end{pmatrix},$$

where $\vartheta_i\,,\ \zeta_i$ are 2×2 fermion matrices which satisfy the following hermiticity conditions

$$\vartheta_1^\dagger = i\vartheta_2^t\sigma_2\,, \quad \vartheta_2^\dagger = -i\vartheta_1^t\sigma_2\,, \quad \zeta_1^\dagger = i\zeta_2^t\sigma_2\,, \quad \zeta_2^\dagger = -i\zeta_1^t\sigma_2\,, \tag{5.15}$$

where σ_i are the Pauli matrices. Computing the Lagrangian (5.14), one finds

$$\begin{aligned}\mathscr{L}_F^{(2)} = \mathrm{tr}\,\big(&2\vartheta_2^t\sigma_2\dot{\vartheta}_1 - \vartheta_1^t\sigma_2\vartheta_1'\sigma_2 + \vartheta_2^t\sigma_2\vartheta_2'\sigma_2 - i\vartheta_2^t\sigma_2\vartheta_1\\ &+ 2\zeta_2^t\sigma_2\dot{\zeta}_1 - \zeta_1^t\sigma_2\zeta_1'\sigma_2 + \zeta_2^t\sigma_2\zeta_2'\sigma_2 - 2i\zeta_2^t\sigma_2\zeta_1 - i\zeta_2^t\sigma_2\zeta_1\sigma_3\big)\,.\end{aligned} \tag{5.16}$$

It is now clear that the last term in (5.16) can be removed by the following fermion redefinition

$$\zeta_1 \to \zeta_1 e^{i\tau\sigma_3/2}\,, \quad \zeta_2 \to \zeta_2 e^{-i\tau\sigma_3/2}\,, \tag{5.17}$$

and the first and the second lines (without the last term) of eq. (5.16) describe eight fermions of mass 1/2 and eight fermions of mass 1, respectively.

The sum of the quadratic bosonic and fermionic actions coincides with the light-cone Green-Schwarz action for type IIA superstrings on the pp-wave background with 24 supersymmetries constructed in [9, 10].

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A. Gamma- and T-matrices

Introduce the following matrices

$$\begin{aligned}
\Gamma^0 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \Gamma^1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \\
\Gamma^2 &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},
\end{aligned} \tag{A.1}$$

These matrices satisfy the Clifford algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$, where $\eta^{\mu\nu}$ is Minkowski metric with signature $(1, -1, -1, -1)$. We also define $\Gamma^5 = -i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$ with the property $(\Gamma^5)^2 = \mathbb{I}$.

The charge conjugation matrix C_4 obeys $(\Gamma^\mu)^t = -C_4\Gamma^\mu C_4^{-1}$ and in the present case it can be chosen as

$$C_4 = i\Gamma^0\Gamma^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{A.2}$$

The Lie algebra $\mathfrak{so}(3,2)$ is generated by the generators $M^{ab} = -M^{ba}$ with $a, b = 0, \ldots, 4$ obeying the following relations

$$[M^{ab}, M^{cd}] = \bar{\eta}^{bc} M^{ad} - \bar{\eta}^{ac} M^{bd} - \bar{\eta}^{bd} M^{ac} + \bar{\eta}^{ad} M^{bc} ,$$

where $\bar{\eta} = \mathrm{diag}(1, -1, -1, -1, 1)$. These generators have the following representation by 4×4 matrices $M^{\mu\nu} = \frac{1}{4}[\Gamma^\mu, \Gamma^\nu] \equiv \Gamma^{\mu\nu}$ and $M^{\mu 4} = \frac{i}{2}\Gamma^\mu$. Such an identification provides an isomorphism $\mathfrak{so}(3,2) \sim \mathfrak{usp}(2,2)$ because in this representation $(M^{ab})^\dagger \Gamma^0 + \Gamma^0 M^{ab} = 0$. The matrices $\Gamma^{\mu\nu}$ generate the Lie algebra $\mathfrak{so}(3,1)$ and they all commute with Γ^5. Finally, $i\Gamma^\mu$ span a space of solutions to the equation $\Omega(A) = -A$ for A restricted to $\mathfrak{usp}(2,2)$.

The stationary subalgebra of the automorphism Ω restricted to the $\mathfrak{so}(6)$ component is determined by the condition

$$[K_6, Y] = 0 , \qquad Y \in \mathfrak{so}(6).$$

The solution to this equation can be parametrized as follows

$$Y = \begin{pmatrix} 0 & y_{12} & y_{24} & -y_{23} & y_{26} & -y_{25} \\ -y_{12} & 0 & y_{23} & y_{24} & y_{25} & y_{26} \\ -y_{24} & -y_{23} & 0 & y_{34} & y_{46} & -y_{45} \\ y_{23} & -y_{24} & -y_{34} & 0 & y_{45} & y_{46} \\ -y_{26} & -y_{25} & -y_{46} & -y_{45} & 0 & y_{56} \\ y_{25} & -y_{26} & y_{45} & -y_{46} & -y_{56} & 0 \end{pmatrix} . \tag{A.3}$$

This is a 9-parametric solution which describes an embedding of the $\mathfrak{u}(3) \subset \mathfrak{so}(6)$.

The space orthogonal to $\mathfrak{u}(3)$ in $\mathfrak{so}(6)$ is spanned by solutions to the following equation

$$K_6 Y = -Y K_6 \tag{A.4}$$

and it provides a parametrization of the coset space $\mathbb{CP}^3$. The general solution to eq. (A.4) is six-parametric and is represented by a matrix

$$Y = \begin{pmatrix} 0 & 0 & y_1 & y_2 & y_3 & y_4 \\ 0 & 0 & y_2 & -y_1 & y_4 & -y_3 \\ -y_1 & -y_2 & 0 & 0 & y_5 & y_6 \\ -y_2 & y_1 & 0 & 0 & y_6 & -y_5 \\ -y_3 & -y_4 & -y_5 & -y_6 & 0 & 0 \\ -y_4 & y_3 & -y_6 & y_5 & 0 & 0 \end{pmatrix} \equiv y_i T_i . \tag{A.5}$$

Here we have introduced the six matrices T_i which are Lie algebra generators of $\mathfrak{so}(6)$ along the $\mathbb{CP}^3$ directions:

$$\begin{aligned} T_1 &= E_{13} - E_{31} - E_{24} + E_{42} , & T_2 &= E_{14} - E_{41} + E_{23} - E_{32} , \\ T_3 &= E_{15} - E_{51} - E_{26} + E_{62} , & T_4 &= E_{16} - E_{61} + E_{25} - E_{52} , \\ T_5 &= E_{35} - E_{53} - E_{46} + E_{64} , & T_6 &= E_{36} - E_{63} + E_{45} - E_{54} , \end{aligned} \tag{A.6}$$

where E_{ij} are the standard matrix unities. The matrices T_i are normalized as follows

$$\mathrm{tr}(T_i T_j) = -4\delta_{ij} . \tag{A.7}$$

The matrices $[T_i, T_j]$ commute with K_6 and they are skew-symmetric.[4] Only 9 of them are independent and they are the generators of $\mathfrak{u}(3)$ inside $\mathfrak{so}(6)$.

Quite remarkably, the matrix (A.5) obeys the following identity

$$Y^3 = -\rho^2 Y\,, \qquad \rho^2 = \sum_{i=1}^{6} y_i^2\,. \tag{A.8}$$

A Lie algebra element parametrizing the coset $\mathrm{AdS}_4 \times \mathbb{CP}^3$ is therefore represented in the form

$$A = \begin{pmatrix} x_\mu \Gamma^\mu & 0 \\ 0 & Y \end{pmatrix}, \qquad Y = y_i T_i\,. \tag{A.9}$$

Thus,

$$A^2 = \begin{pmatrix} x^2 \mathbb{I} & 0 \\ 0 & Y^2 \end{pmatrix}, \qquad A^3 = \begin{pmatrix} x^2 x_\mu \Gamma^\mu & 0 \\ 0 & Y^3 \end{pmatrix} = \begin{pmatrix} x^2 x_\mu \Gamma^\mu & 0 \\ 0 & -\rho^2 Y \end{pmatrix},$$

where $x^2 = x_\mu x_\nu \eta^{\mu\nu}$. Obviously,

$$\mathrm{str} A^2 = 4x^2 + 4y^2\,, \qquad \mathrm{str}\Sigma A^2 = 4x^2 - 4y^2 \tag{A.10}$$

and as the result we find the following characteristic equation

$$A^3 = \frac{1}{8}\,\mathrm{str}(\Sigma A^2)\, A + \frac{1}{8}\,\mathrm{str}(A^2)\, \Sigma A\,, \tag{A.11}$$

for a matrix Lie algebra element A parametrizing the space $\mathrm{AdS}_4 \times \mathbb{CP}^3$.

B. Parametrizations of $\mathbb{CP}^3$

An SO(6) matrix parametrizing the coset space SO(6)/U(3), and therefore $\mathbb{CP}^3$, can be obtained by exponentiating the generic element (A.5). The matrix exponent can be easily computed by using the identity (A.8), and gives a generic representative of the coset in the following form

$$g = e^Y = I + \frac{\sin\rho}{\rho} Y + \frac{1 - \cos\rho}{\rho^2} Y^2\,. \tag{B.1}$$

[4]The anti-commutators $\{T_i, T_j\}$ commute with K_6 as well. As the consequence, all T_{ij} are symmetric. Not all the matrices $\{T_i, T_j\}$ are independent. In particular,

$$\{T_1, T_2\} = 0\,, \quad \{T_3, T_4\} = 0\,, \quad \{T_5, T_6\} = 0$$

which can be verified by a direct calculation. From the remaining matrices $\{T_i, T_j\}$ only six are independent. One can choose, for instance,

$$\{T_1, T_4\}\,, \quad \{T_2, T_4\}\,, \quad \{T_1, T_6\}\,, \quad \{T_2, T_6\}\,, \quad \{T_3, T_6\}\,, \quad \{T_4, T_6\}\,.$$

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The formula (B.1) suggests to parametrize $\mathbb{CP}^3$ by means of the spherical coordinates

$$\begin{aligned} y_1 + iy_2 &= \rho\, \sin\theta \cos\frac{\alpha_1}{2}\, e^{\frac{i}{2}(\alpha_2+\alpha_3)+i\phi}\,, \\ y_3 + iy_4 &= \rho\, \sin\theta \sin\frac{\alpha_1}{2}\, e^{-\frac{i}{2}(\alpha_2-\alpha_3)+i\phi}\,, \\ y_5 + iy_6 &= \rho\, \cos\theta\, e^{i\phi}\,. \end{aligned} \tag{B.2}$$

This provides an explicit parametrization of $\mathbb{CP}^3$ which can be used to find the Fubini-Study metric on $\mathbb{CP}^3$. To this end, by noting that $K_6 g = g^{-1}K_6$ we first compute

$$-2A^{(2)} = g^{-1}dg + K_6 g^{-1} dg K_6 = g^{-1}dg + dg g^{-1}\,.$$

Then, the Fubini-Study metric on $\mathbb{CP}^3$ is given by the following formula

$$\begin{aligned} ds^2_{\mathbb{CP}^3} &= -\frac{1}{4}\mathrm{tr}\left(A^{(2)}\right)^2 = \frac{1}{8}\mathrm{tr}\left(dgdg^t - g^t g^t dgdg\right) = \frac{1}{16}\mathrm{tr}\left(d(g^2)d(g^2)^t\right) \\ &= d\rho^2 + \frac{1}{4}\sin^2 2\rho\left(d\phi + \frac{1}{2}\sin^2\theta\,(d\alpha_3 + d\alpha_2\cos\alpha_1)\right)^2 + \sin^2\rho\, ds^2_{\mathbb{CP}^2}\,, \end{aligned} \tag{B.3}$$

where

$$ds^2_{\mathbb{CP}^2} = d\theta^2 + \frac{1}{4}\sin^2\theta\left(d\alpha_1^2 + \sin^2\alpha_1 d\alpha_2^2 + \cos^2\theta\,(d\alpha_3 + \cos\alpha_1\, d\alpha_2)^2\right)$$

is the Fubini-Study metric on $\mathbb{CP}^2$.

Note also that $\mathbb{CP}^3$ can be also parametrized by means of the following matrices

$$G = gK_6 g^t = g^2 K_6\,, \quad G^t = -G\,, \quad GG^t = I\,, \tag{B.4}$$

and therefore $\mathbb{CP}^3$ can be identified with the intersection of skew-symmetric and orthogonal matrices. In terms of the matrix G the Fubini-Study metric on $\mathbb{CP}^3$ is given by the following simple formula

$$ds^2_{\mathbb{CP}^3} = \frac{1}{16}\mathrm{tr}\, dGdG^t\,. \tag{B.5}$$

It is well known that the Fubini-Study metric on $\mathbb{CP}^3$ can be also written in the form

$$ds^2_{\mathbb{CP}^3} = \frac{d\bar{w}_i dw_i}{1+|w|^2} - \frac{d\bar{w}_i w_i \bar{w}_j dw_j}{(1+|w|^2)^2}\,, \qquad |w|^2 = \bar{w}_k w_k\,. \tag{B.6}$$

The three complex inhomogeneous coordinates w_i are related to the six real coordinates y_i as follows

$$\begin{aligned} &\sin\rho = \frac{|w|}{\sqrt{1+|w|^2}}\,, \quad \cos\rho = \frac{1}{\sqrt{1+|w|^2}}\,, \quad \sin 2\rho = \frac{2|w|}{1+|w|^2}\,, \quad 1-\cos 2\rho = \frac{2|w|^2}{1+|w|^2}\,, \\ &\frac{|w|}{\rho}(y_1+iy_2) = w_1\,, \quad \frac{|w|}{\rho}(y_3+iy_4) = w_2\,, \quad \frac{|w|}{\rho}(y_5+iy_6) = w_3\,. \end{aligned} \tag{B.7}$$

Then, the coset representative g takes the form

$$g = I + \frac{1}{\sqrt{1+|w|^2}}(W + \overline{W}) + \frac{\sqrt{1+|w|^2}-1}{|w|^2\sqrt{1+|w|^2}}(W\overline{W} + \overline{W}W)\,, \tag{B.8}$$

where

$$
\begin{aligned}
W &= w_i \mathcal{T}_i\,, \quad \mathcal{T}_1 = \frac{1}{2}(T_1 - iT_2)\,, \quad \mathcal{T}_2 = \frac{1}{2}(T_3 - iT_4)\,, \quad \mathcal{T}_3 = \frac{1}{2}(T_5 - iT_6)\,, \\
W &= \bar{w}_i \overline{\mathcal{T}}_i\,, \quad \overline{\mathcal{T}}_1 = \frac{1}{2}(T_1 + iT_2)\,, \quad \overline{\mathcal{T}}_2 = \frac{1}{2}(T_3 + iT_4)\,, \quad \overline{\mathcal{T}}_3 = \frac{1}{2}(T_5 + iT_6)\,,
\end{aligned}
\tag{B.9}
$$

and we took into account that $W^2 = 0$ for any set of w_i. Computing g^2 we get the following simple formula

$$
g^2 = I + \frac{2}{1+|w|^2}(W + \overline{W}) + \frac{2}{1+|w|^2}(W\overline{W} + \overline{W}W)\,, \tag{B.10}
$$

which can be used to find G and verify (B.6).

The $\mathbb{CP}^3$ metric can be written in terms of the four homogeneous coordinates z_a

$$
ds^2_{\mathbb{CP}^3} = \frac{d\bar{z}_a dz_a}{\bar{z}_c z_c} - \frac{d\bar{z}_a z_a \bar{z}_b dz_b}{(\bar{z}_c z_c)^2}\,, \tag{B.11}
$$

which is the standard form of the Fubini-Study metric. Inhomogeneous coordinates w_i are related to z_a as follows

$$
w_i = \frac{z_i}{z_4}\,, \tag{B.12}
$$

and the metric (B.11) obviously reduces to (B.6) if $z_4 = 1$.

It is clear from (B.6) that there are 3 commuting isometry directions corresponding to multiplying w_i by a phase $w_i \to e^{i\alpha} w_i$.

References

[1] J.H. Schwarz, *Superconformal Chern-Simons theories*, *JHEP* **11** (2004) 078 [hep-th/0411077].

[2] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D 75** (2007) 045020 [hep-th/0611108]; *Gauge symmetry and supersymmetry of multiple M2-branes*, *Phys. Rev.* **D 77** (2008) 065008 [arXiv:0711.0955];
A. Gustavsson, *Algebraic structures on parallel M2-branes*, arXiv:0709.1260;
M. Van Raamsdonk, *Comments on the Bagger-Lambert theory and multiple M2-branes*, *JHEP* **05** (2008) 105 [arXiv:0803.3803];
J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, *Supersymmetric Yang-Mills theory from lorentzian three-algebras*, arXiv:0806.0738;
M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence*, arXiv:0806.1519.

[3] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, arXiv:0806.1218.

[4] J.A. Minahan and K. Zarembo, *The Bethe ansatz for superconformal Chern-Simons*, arXiv:0806.3951.

[5] E. Abdalla, M. Forger and M. Gomes, *On the origin of anomalies in the quantum nonlocal charge for the generalized nonlinear σ-models*, *Nucl. Phys.* **B 210** (1982) 181;
E. Abdalla, M. Forger and A. Lima Santos, *Nonlocal charges for nonlinear σ-models on Grassmann manifolds*, *Nucl. Phys.* **B 256** (1985) 145;
J.M. Evans, D. Kagan and C.A.S. Young, *Non-local charges and quantum integrability of σ-models on the symmetric spaces* $\mathrm{SO}(2n)/\mathrm{SO}(n)\times\mathrm{SO}(n)$ *and* $\mathrm{Sp}(2n)/\mathrm{Sp}(n)\times\mathrm{Sp}(n)$, *Phys. Lett.* **B 597** (2004) 112 [hep-th/0404003].

[6] S. Watamura, *Spontaneous compactification and Cp(N):* $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, $\sin^2(\theta_w)$, G_3/G_2 *and* $\mathrm{SU}(3)$ *triplet chiral fermions in four-dimensions*, *Phys. Lett.* **B 136** (1984) 245.

[7] B. de Wit, K. Peeters, J. Plefka and A. Sevrin, *The M-theory two-brane in $AdS_4\times S^7$ and $AdS_7\times S^4$*, *Phys. Lett.* **B 443** (1998) 153 [hep-th/9808052].

[8] R.R. Metsaev and A.A. Tseytlin, *Type IIB superstring action in $AdS_5\times S^5$ background*, *Nucl. Phys.* **B 533** (1998) 109 [hep-th/9805028].

[9] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on pp-wave*, *Nucl. Phys.* **B 644** (2002) 128 [hep-th/0208029].

[10] S.-J. Hyun and H.-J. Shin, *$N=(4,4)$ type IIA string theory on pp-wave background*, *JHEP* **10** (2002) 070 [hep-th/0208074].

[11] T. Nishioka and T. Takayanagi, *On type IIA Penrose limit and $N=6$ Chern-Simons theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391].

[12] D. Gaiotto, S. Giombi and X. Yin, *Spin chains in $N=6$ superconformal Chern-Simons-matter theory*, arXiv:0806.4589.

[13] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5\times S^5$ superstring*, *Phys. Rev.* **D 69** (2004) 046002 [hep-th/0305116].

[14] G. Arutyunov and S. Frolov, *Integrable Hamiltonian for classical strings on $AdS_5\times S^5$*, *JHEP* **02** (2005) 059 [hep-th/0411089].

[15] S. Frolov, J. Plefka and M. Zamaklar, *The $AdS_5\times S^5$ superstring in light-cone gauge and its Bethe equations*, *J. Phys.* **A 39** (2006) 13037 [hep-th/0603008].

[16] N. Beisert, *The* $\mathrm{SU}(2|2)$ *dynamic S-matrix*, hep-th/0511082.

[17] G. Arutyunov, S. Frolov, J. Plefka and M. Zamaklar, *The off-shell symmetry algebra of the light-cone $AdS_5\times S^5$ superstring*, *J. Phys.* **A 40** (2007) 3583 [hep-th/0609157].

[18] V.A. Kazakov, A. Marshakov, J.A. Minahan and K. Zarembo, *Classical/quantum integrability in AdS/CFT*, *JHEP* **05** (2004) 024 [hep-th/0402207];
N. Beisert, V.A. Kazakov, K. Sakai and K. Zarembo, *The algebraic curve of classical superstrings on $AdS_5\times S^5$*, *Commun. Math. Phys.* **263** (2006) 659 [hep-th/0502226].

[19] G. Arutyunov, S. Frolov and M. Staudacher, *Bethe ansatz for quantum strings*, *JHEP* **10** (2004) 016 [hep-th/0406256].

[20] R.A. Janik, *The $AdS_5\times S^5$ superstring worldsheet S-matrix and crossing symmetry*, *Phys. Rev.* **D 73** (2006) 086006 [hep-th/0603038].

[21] N. Beisert, R. Hernandez and E. Lopez, *A crossing-symmetric phase for $AdS_5\times S^5$ strings*, *JHEP* **11** (2006) 070 [hep-th/0609044];
N. Beisert, B. Eden and M. Staudacher, *Transcendentality and crossing*, *J. Stat. Mech.* (2007) P01021 [hep-th/0610251].

[22] M.B. Green and J.H. Schwarz, *Covariant description of superstrings*, *Phys. Lett.* **B 136** (1984) 367.

[23] M. Cvetič, H. Lü, C.N. Pope and K.S. Stelle, *T-duality in the Green-Schwarz formalism and the massless/massive IIA duality map*, *Nucl. Phys.* **B 573** (2000) 149 [hep-th/9907202].

[24] R. Roiban and W. Siegel, *Superstrings on $AdS_5 \times S^5$ supertwistor space*, *JHEP* **11** (2000) 024 [hep-th/0010104].

[25] A.K. Das, J. Maharana, A. Melikyan and M. Sato, *The algebra of transition matrices for the $AdS_5 \times S^5$ superstring*, *JHEP* **12** (2004) 055 [hep-th/0411200];
A.K. Das, A. Melikyan and M. Sato, *The algebra of flat currents for the string on $AdS_5 \times S^5$ in the light-cone gauge*, *JHEP* **11** (2005) 015 [hep-th/0508183].

[26] L.F. Alday, G. Arutyunov and A.A. Tseytlin, *On integrability of classical superstrings in $AdS_5 \times S^5$*, *JHEP* **07** (2005) 002 [hep-th/0502240].

[27] V.V. Serganova, *Automorphisms of simple Lie superalgebras*, *Math. USSR Izv.* **24** (1985) 539.

[28] J.A. de Azcarraga and J. Lukierski, *Supersymmetric particles with internal symmetries and central charges*, *Phys. Lett.* **B 113** (1982) 170.

[29] W. Siegel, *Hidden local supersymmetry in the supersymmetric particle action*, *Phys. Lett.* **B 128** (1983) 397.

[30] I.N. McArthur, *κ-symmetry of Green-Schwarz actions in coset superspaces*, *Nucl. Phys.* **B 573** (2000) 811 [hep-th/9908045].

[31] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051].

[32] L.F. Alday, G. Arutyunov and S. Frolov, *Green-Schwarz strings in TsT-transformed backgrounds*, *JHEP* **06** (2006) 018 [hep-th/0512253].

[33] L.F. Alday, G. Arutyunov and S. Frolov, *New integrable system of 2 dim fermions from strings on $AdS_5 \times S^5$*, *JHEP* **01** (2006) 078 [hep-th/0508140].

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